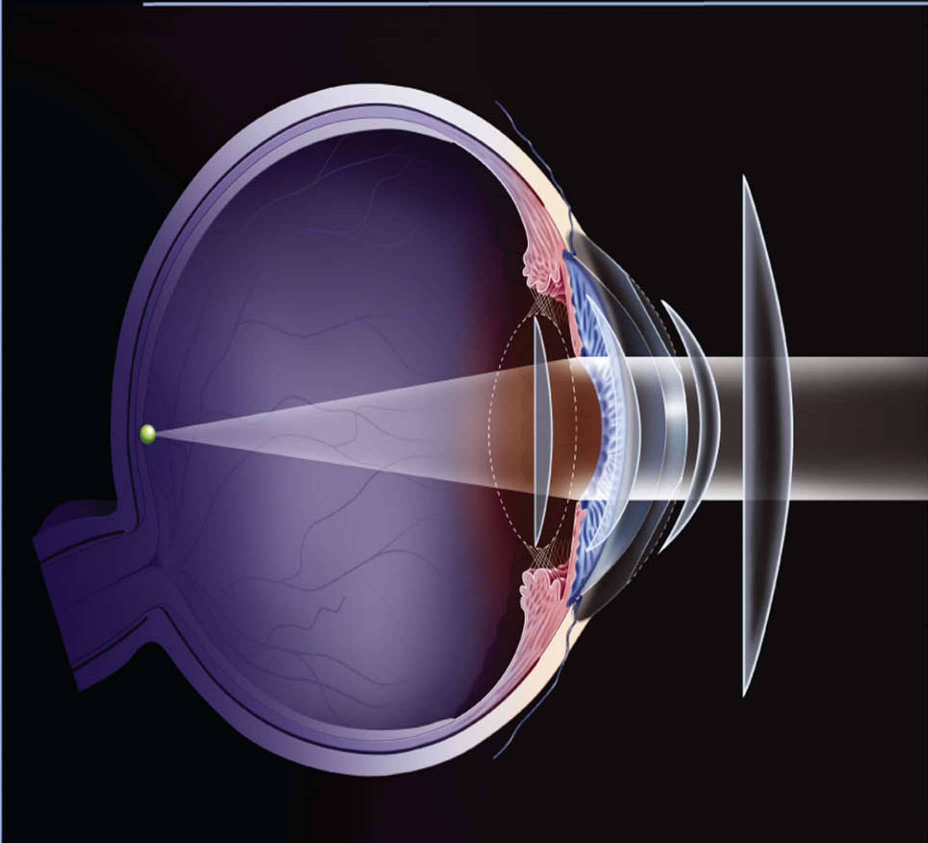
SECOND EDITION

# GEOMETRICAL AND VISUAL OPTICS

## A Clinical Introduction



STEVEN H. SCHWARTZ

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## A Clinical Introduction

SECOND EDITION

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# Preface

*...I strove that not one hour*
*Should idly pass. My eyes and mind took pride*
*In sacred Optics.*[1]

Jan Vredeman De Vries
1527–c.1604

This book is intended as an approachable and appropriately rigorous introduction to geometrical and visual optics. It is meant to be a concise and learner-friendly resource for clinicians as they study optics for the first time and as they subsequently prepare for licensing examinations. The emphasis is on those optical concepts and problem-solving skills that underlie contemporary clinical eye care.

Because of its clinical utility, a vergence approach is stressed. While formulae are an inevitable part of optics, an attempt has been made to keep these to a minimum by emphasizing underlying concepts. Plentiful schematic figures and clinical examples are used to engage reader interest and foster understanding. Every effort is made to provide the reader with an intuitive and clinical sense of optics that will allow him or her to effectively care for patients.

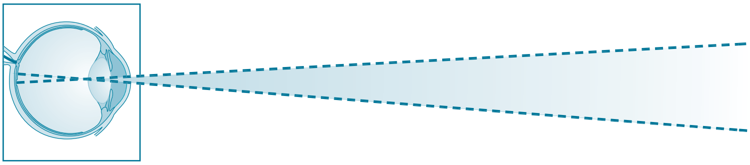
To develop facility in geometrical and visual optics, it is necessary to solve problems. Each chapter includes self-assessment problems of varying complexity with detailed worked-out solutions given at the end of the book. These problems are an integral part of the text.

The second edition has several new features intended to improve student learning. Figures have been upgraded and are now in color. Summaries, sample problems, and tables within chapters are color highlighted. At the conclusion of each chapter, there is a brief summary and list of formulae. New self-assessment problems have been added to many chapters. To meet student demand for additional self-assessment tools, two comprehensive practice examinations (with answers) are

1. Vredeman De Vries, Jan (1604). *Studies in Perspective.* Republished in 2010 by Dover Publications, Inc.

included. Throughout the book, sections have been rewritten and reorganized to make the material less intimidating and more comprehensible.

It was my good fortune to be able to call upon knowledgeable and generous colleagues to review all or portions of earlier drafts of the second edition. The thoughtful input of Drs. Kathy Aquilante, Ian Bailey, Cliff Brooks, Jay Cohen, Geoffrey Goodfellow, Ralph Gundel, John Mark Jackson, Phil Kruger, Cristina Llerena Law, Jeff Rabin, Alan Reizman, Jie (Jason) Shen, and Frank Spors is greatly appreciated. Any shortcomings of the book are, of course, entirely my responsibility.



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# Basic Terms and Concepts

While we may think we're aware of what's going on around us, we're missing out on quite a bit. Our eyes are continuously bombarded by electromagnetic (EM) radiation, but as illustrated in Figure 1-1, we see only a small fraction of it. The remainder of the EM spectrum, including x-rays, ultraviolet (UV) and infrared radiation, and radar and radio waves, is invisible.

EM radiation is specified by its wavelength. As can be seen in Figure 1-2, wavelength and frequency are inversely proportional—as the wavelength increases, frequency decreases (and vice versa).[1] They are related to each other as follows:

$$f = \frac{v}{\lambda}$$

where $f$ is the frequency of the EM radiation, $v$ is the speed of the EM radiation, and $\lambda$ is the wavelength of the EM radiation.

Visible radiation—light—ranges from about 380 to 700 nm.[2] This region of the spectrum is absorbed by the retinal photopigments, setting in motion a complex chain of events that result in vision.[3]

EM radiation is emitted in discrete packages of energy referred to as **photons** or **quanta**. The amount of energy in a photon is given by

$$E = hf$$

where $E$ is the amount of energy per photon and $h$ is Planck's constant.

1. As light travels from a less dense material, such as air, to a more dense material, such as water, its frequency does not change, but its speed and wavelength decrease.
2. One nanometer is equal to $10^{-9}$ m.
3. For a basic introduction to visual processes see Schwartz SH. *Visual Perception: A Clinical Orientation*. 4th ed. New York: McGraw-Hill; 2010.

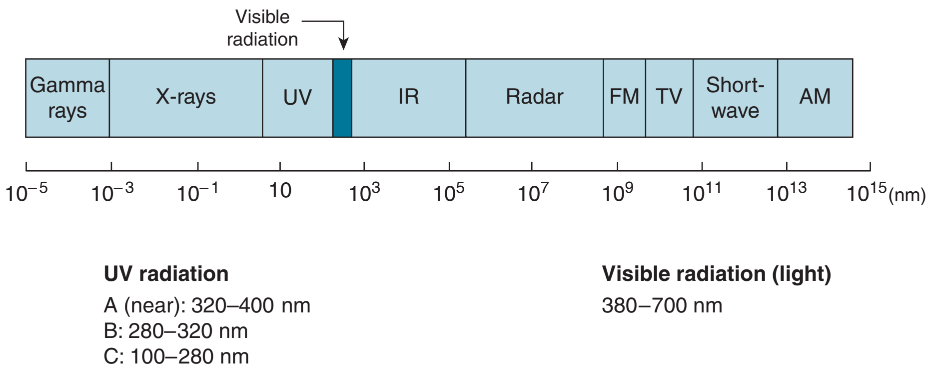


**Figure 1-1.** Light (visible radiation), a small portion of the EM spectrum, ranges from about 380 to 700 nm. UV radiation, which because of its high energy contributes to the development of various ocular and skin conditions, can be classified as UVA, UVB or UVC. (*Reproduced with permission from Schwartz SH.* Visual Perception: A Clinical Orientation. *4th ed. http://www.accessmedicine.com. Copyright © 2010 McGraw-Hill Education. All rights reserved.*)

By substitution, we have:

$$E = \frac{hv}{\lambda}$$

As the wavelength decreases, the amount of energy per photon increases. For this reason, the absorption of short-wavelength radiation by body tissues is typically more damaging than the absorption of longer-wavelength radiation. The

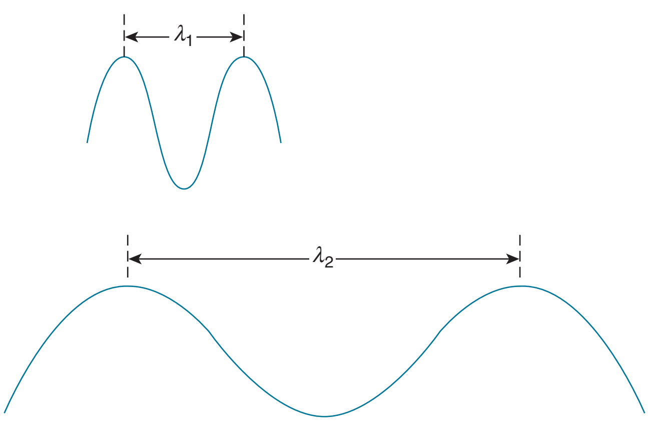


**Figure 1-2.** Wavelength ($\lambda$) and frequency are inversely proportional to each other. (*Adapted with permission from Schwartz SH.* Visual Perception: A Clinical Orientation. *4th ed. http://www.accessmedicine.com. Copyright © 2010 McGraw-Hill Education. All rights reserved.*)

development of skin cancer, pinguecula, pterygium, photokeratitis, cataracts, and age-related macular degeneration has been linked to exposure to short-wavelength, high-energy UV radiation. Ocular exposure can be minimized by use of spectacles that block these rays and headgear (hats, visors) that protect the eye and its adnexa.

Longer-wavelength UV radiation may be categorized as either UVB, which ranges from 280 to 320 nm, or UVA (320–400 nm). UVB is absorbed by the skin epidermis resulting in sunburns. This radiation is most abundant during the summer months. In comparison, UVA, which penetrates deeper into the skin and is absorbed by the dermis, is present all year long. Accumulated damage to the dermis results in wrinkling of the skin and is responsible for commuter aging—wrinkling in areas that are exposed to sunlight (e.g., neck and back of hands) while driving to work. Both UVB and UVA have been associated with skin cancer.

## SOURCES, LIGHT RAYS, AND PENCILS

For the study of geometrical and visual optics, we are interested primarily in the wave nature of light rather than its quantal nature. Figure 1-3 shows that a **point source**[4] of light, such as a star, emits concentric waves of light in much the same

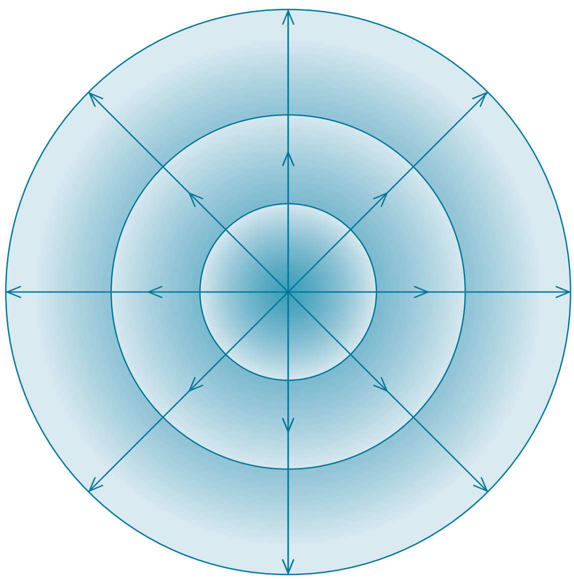

**Figure 1-3.** A point source of light emits concentric waves of light in much the same way a pebble dropped into a quiet pond of water produces waves of water. Light rays, represented by arrows, are perpendicular to the wavefronts.

4. The size of a point source approaches zero—it is infinitely small.

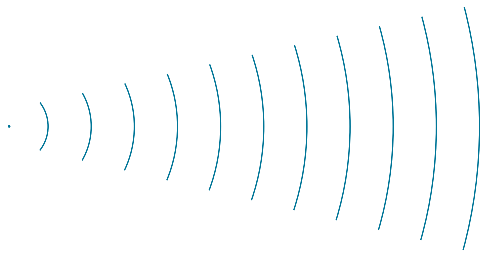

**Figure 1-4.** The curvature of wavefronts becomes less as the distance from the point source increases. They are arcs of a circle whose center is the point source. At infinity, the wavefronts are flat.

way that a pebble dropped into a quiet pond of water generates waves of water. The peaks of the waves are called wavefronts. Think of them as circles with radii equal to the distance from the point source.

Let's look at this in more detail. Figure 1-4 shows wavefronts traveling from left to right. Consider these to be arcs of a circle whose center is the point source. As you can see, the curvature of these wavefronts decreases as the distance from the source increases. An arc with a longer radius is flatter than one with a shorter radius. At infinity (where the radius of the arc is infinity), the wavefronts are flat.

Note that direction of movement of the wavefronts in Figure 1-3 is represented by arrows—commonly called **light rays**—that are perpendicular to the wavefronts. A bundle of rays is called a **pencil**. As illustrated in Figure 1-5, the light rays that form a pencil can be diverging, converging, or parallel. A **diverging pencil** is produced by a point source of light, such as a star. When light rays are focused at a point, they create a **converging pencil**. A converging optical system (e.g., a magnifying lens) is required to create converging light. An object located infinitely far away forms a **parallel pencil** because, as we've seen in Figure 1-4, the wavefronts are flat (which means that the rays perpendicular to these wavefronts must be parallel to each other).

An **extended source**, such as the arrow in Figure 1-6, is composed of an infinite number of point sources. Diverging light rays emerge from each of the point sources.

## VERGENCE

When it comes to understanding and solving clinical optical problems, the concept of vergence goes a long way. At this point, I'll provide some working definitions that will get you going. Once we start looking at optical problems in subsequent chapters, vergence will become second nature to you (I hope!).

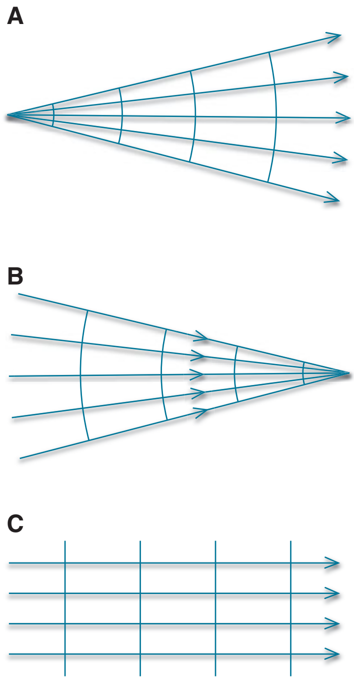


**Figure 1-5. A.** A diverging pencil of light rays emerges from a point source. **B.** A converging pencil of light rays is focused at a point. **C.** An object located at infinity produces a parallel pencil of light rays. Note that the light rays are perpendicular to the wavefronts.

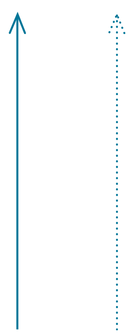

**Figure 1-6.** An extended object, such as an arrow, may be considered to consist of an infinite number of point sources. Each point emits diverging light rays.

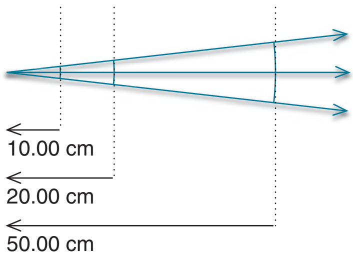


**Figure 1-7.** Diverging light rays have negative vergence. At distances of 10.00, 20.00, and 50.00 cm, the vergence is –10.00, –5.00, and –2.00 D, respectively. The magnitude of the vergence (ignoring the sign) *decreases* as the distance to the source *increases*.

Vergence is a way to quantify the curvature of a wavefront. For point sources, curvature is greatest near the source and diminishes with distance from the source. The more curved a wavefront is, the greater its vergence. Likewise, the less curved it is, the less its vergence.

**When solving optical problems, the vergence of diverging light is always—yes, *always*—labeled with a negative sign. The amount of divergence is quantified by taking the reciprocal of the distance *to* a point source.** To arrive at the correct units for vergence—diopters (D)—the distance must be in meters. This may sound more difficult than it is. Figure 1-7, which gives vergence at three distances from a point source, should help. At 10.00 cm the vergence is –10.00 D, at 20.00 cm it is –5.00 D, and at 50 cm it is –2.00 D. In each case, we convert the distance to meters, take the reciprocal, and then label the vergence as negative to indicate that the light is diverging.[5] **Note that the magnitude of the vergence (ignoring the sign) is greatest close to the source and diminishes as the distance increases.**

As we mentioned previously, not all light is diverging. An optical system, such as a magnifying lens, can produce converging light. **To solve optical problems, the vergence of converging light is always—yes, *always*—labeled with a plus sign. It is quantified by taking the reciprocal of the distance (in meters) *to* the point where the light is focused.** Consider Figure 1-8, which shows light converging to a point focus. The vergence measured at distances of 10.00, 20.00, and 50.00 cm from this focus point is +10.00, +5.00, and +2.00 D, respectively. **Note that the vergence is greatest close to the focus point and decreases as the distance increases.**

5. In Chapter 3, we'll learn that when light rays are in a substance other than air, the vergence is increased. We'll talk more about this later.

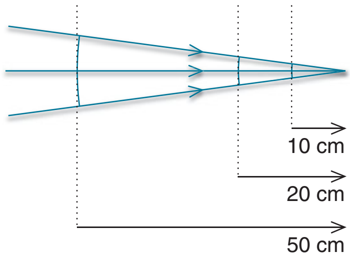


**Figure 1-8.** Converging light rays have positive vergence. At the distances of 10.00, 20.00, and 50.00 cm, the vergence is +10.00, +5.00, and +2.00 D, respectively. As the distance to the point of focus *increases*, convergence *decreases*.

What is the vergence of a light source located infinitely far away? The wavefronts are flat—they have no curvature—making the vergence equal to zero. Thinking of it in quantitative terms, the reciprocal of the distance to the object (infinity) is zero. Or think of it this way: since the light rays are neither diverging nor converging, the vergence is zero. For clinical purposes, we normally consider distances greater than 20 ft (or 6 m) as infinitely far away.

## REFRACTION AND SNELL'S LAW

The velocity of light depends on the medium in which it is traveling. Light travels more slowly in an optically dense medium, such as glass, than it does in a less dense medium, such as air. The degree to which an optical medium slows the velocity of light is given by its refractive index, which is the ratio of the speed of light in a vacuum to its speed in the medium. Refractive indices of materials commonly encountered in clinical practice are given in Table 1-1.

**TABLE 1-1. REFRACTIVE INDICES OF COMMON MATERIALS**

| Material | Refractive Index |
|---|---|
| Air | 1.000 |
| Water | 1.333 |
| Ophthalmic plastic (CR39) | 1.498 |
| Crown glass | 1.523 |
| Trivex | 1.532 |
| Polycarbonate | 1.586 |
| Essilor Airwear (plastic) | 1.59 |
| Essilor Thin & Lite (plastic) | 1.67 or 1.74 |

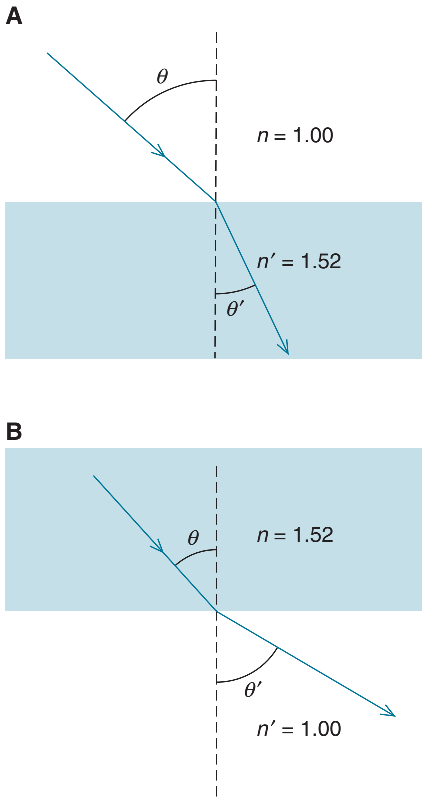


**Figure 1-9. A.** A light ray entering a denser medium is refracted *toward* the normal. **B.** A ray entering a rarer medium is refracted *away* from the normal.

The change in velocity that occurs as light travels from one optical medium into another may cause a light ray to deviate from its original direction, a phenomenon referred to as **refraction**. Figure 1-9A illustrates the refraction that occurs when light traveling in air strikes a glass surface at an angle, $\theta$, as measured with respect to the normal to the surface. The decrease in velocity causes the ray to change its direction. In this case, the light ray is refracted so that the angle made with the normal to the surface is decreased to $\theta'$.

This illustrates a general rule that you should memorize—when a light ray traveling in a material with a low index of refraction (an optically rarefied medium) enters a material with a higher index of refraction (an optically denser medium), the light ray is refracted *toward* (i.e., bent toward) the normal to the surface.

What occurs when light traveling in an optically dense medium enters one that is less dense? As can be seen in Figure 1-9B, the increase in velocity causes the light ray to be deviated away from the normal. Again, this is a handy fact to memorize.

It can be useful to quantify the refraction that occurs as light travels from one medium, which we'll call the **primary medium**, into another medium, which is called the **secondary medium**. Snell's law, which is given below, allows us to do so:

$$n(\sin\theta) = n'(\sin\theta')$$

where $n$ is the index of refraction of the primary medium, $n'$ is the index of refraction of the secondary medium, $\theta$ is the angle of incidence (with respect to the normal), and $\theta'$ is the angle of refraction (with respect to the normal).

Let's do a problem. *For a light ray traveling from air to crown glass, the angle of incidence is 20.00 degrees. What is the angle of refraction?*

In this and almost all optical problems, it's a very good idea to draw a diagram. Figure 1-10 shows a light ray striking the glass surface such that it makes an angle of 20 degrees with the normal to the surface. Before doing the calculation, we know that the light ray is refracted toward the normal. How do we know this?

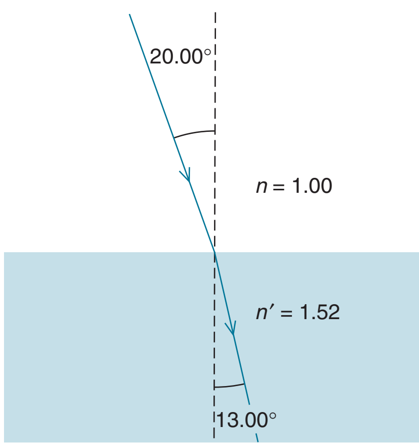


**Figure 1-10.** For a light ray that strikes a crown glass surface at an angle of 20.00 degrees, the angle of refraction is 13.00 degrees.

As we mentioned earlier, when a light ray travels into a material with a higher index of refraction, it is deviated toward the normal. Snell's law allows us to determine the angle of refraction as follows:

$$n(\sin\theta) = n'(\sin\theta')$$

$$(1.00)(\sin 20.00°) = (1.52)(\sin\theta')$$

$$\theta' = 13.00°$$

Let us look at another example. *A light ray travels from a diamond (n = 2.42) into air. What is the angle of refraction if the angle of incidence is 5.00 degrees?*

Because the light ray is entering a medium with a lower index of refraction, we know that it is refracted away from the normal, as illustrated in Figure 1-11A. The angle of refraction is calculated using Snell's law:

$$n(\sin\theta) = n'(\sin\theta')$$

$$(2.42)(\sin 5.00°) = (1.00)(\sin\theta')$$

$$\theta' = 12.18°$$

An interesting situation occurs when the angle of incidence for the light ray traveling from diamond to air is increased to 24.40 degrees. According to Snell's law:

$$n(\sin\theta) = n'(\sin\theta')$$

$$(2.42)(\sin 24.40°) = (1.00)(\sin\theta')$$

$$\theta' \approx 90°$$

Figure 1-11B shows that the refracted ray is approximately parallel to the surface. What happens if the angle of incidence is further increased? As can be seen in Figure 1-11C, when the angle of incidence exceeds 24.40 degrees, which is referred to as the **critical angle**, the light ray does not emerge from the material—it undergoes a phenomenon referred to as **total internal reflection**.

Total internal reflection prevents the clinician from seeing the structures that constitute the angle of the eye—structures that must be assessed in glaucoma and other diseases—unless a special instrument called a goniolens is used. Figure 1-12 shows that the goniolens reduces total internal reflection, allowing the angle of the eye to be visualized.

## SUMMARY

A bundle of light rays—commonly referred to as a pencil—can be diverging, converging, or parallel. The amount of divergence or convergence, which we call vergence, can be quantified by taking the reciprocal of the distance (in meters) to the

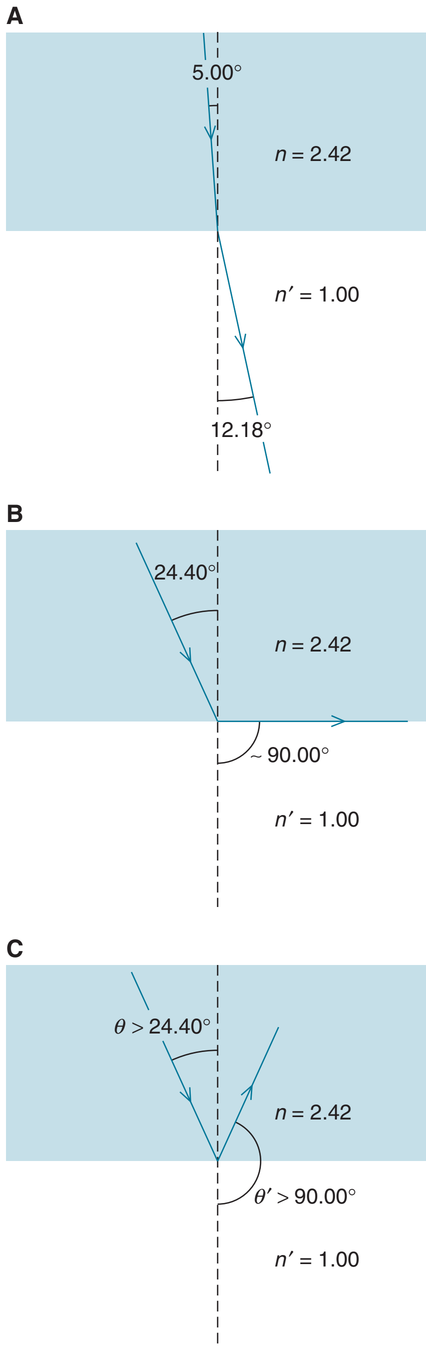


**Figure 1-11.** A light ray travels from a diamond toward air. **A.** For an angle of incidence of 5.00 degrees, the angle of refraction is 12.18 degrees. **B.** If the angle of incidence is 24.40 degrees, the angle of refraction is about 90.00 degrees. The refracted ray is approximately parallel to the surface. **C.** When the angle of incidence exceeds the critical angle (~24.40 degrees), the light ray undergoes total internal reflection.

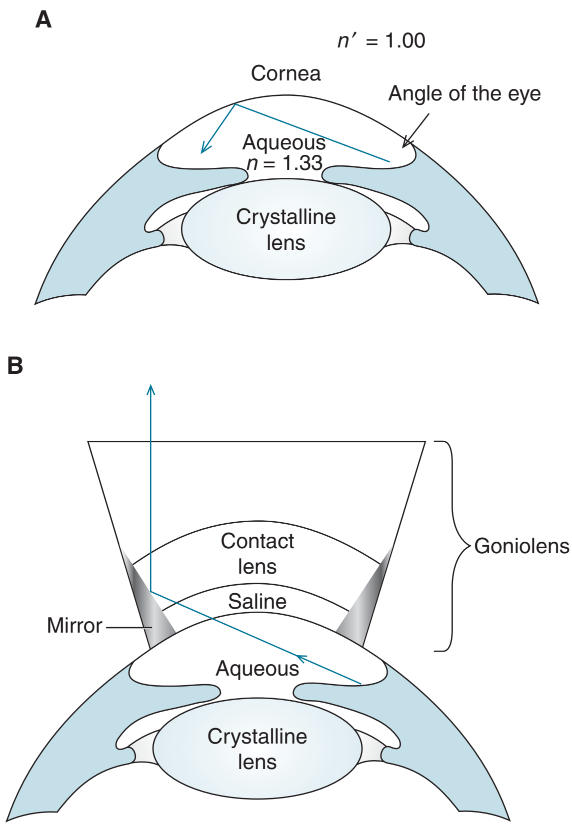


**Figure 1-12. A.** A light ray emerging from the angle of the eye undergoes total internal reflection if the angle of incidence (at the cornea) exceeds ~49 degrees. (The light ray is traveling from the higher index aqueous humor toward the lower index air.) Total internal reflection prevents the doctor from examining the angle unless he or she uses a device referred to as a *goniolens*. **B.** A goniolens allows visualization of the angle of the eye by reducing total internal reflection. A saline-like fluid is placed between the cornea and the contact lens that constitutes the front of the goniolens. Since the saline and the aqueous humor have about the same index of refraction, total internal reflection is substantially reduced. This allows rays emerging from the angle to pass out of the eye. They are reflected by a mirror in the goniolens that the doctor looks into, allowing him or her to see the structures that constitute the angle. (This diagram is a simplification.)

point of divergence or convergence. Diverging light is specified with a minus sign and converging light with a plus sign.

The direction of a light ray can change when it travels from one medium to another. The magnitude of this change is given by Snell's law, which is probably the most fundamental law of geometrical optics.

## KEY FORMULA

*Snell's law:*

$n(\sin\theta) = n'(\sin\theta')$

## SELF-ASSESSMENT PROBLEMS

1. A ray of light emerges from a pond of water at an angle of 45 degrees to the normal. What angle did the incident ray make with the normal?
2. A ray of light is incident upon a pond of water that is 2.0 m deep. If the angle of incidence is 25 degrees, by how many centimeters is the ray deviated as it travels through the pond?
3. A crown glass slab, 75.00 cm thick, is surrounded by air. A ray of light makes an angle of 30 degrees to the normal at the front surface of the slab. At what angle (to the normal) does the ray emerge from the slab?
4. What is the smallest angle of incidence that will result in total internal reflection for a light ray traveling from a high-index glass (index of 1.72) to air?
5. What is the critical angle for a diamond surrounded by water?

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